An Easie Demonstration of the Analogy of the Logarithmick Tangents to the Meridian Line or sum of the Secants: with various Methods for computing the same to the utmost Exactness, by E. Halley.

IT is now near 100 Years since our Worthy Countryman Mr. *Edward Wright* published his *Correction of Errors in Navigation*, a Book well deserving the perusal of all such as design to use the Sea. Therein he considers the Course of a Ship on the Globe, stearing obliquely to the Meridian; and having shewn, that the *Departure* from the Meridian, is in all cases less than the *Difference* of *Longitude*, in the *ratio* of *Radius* to the *secant* of the *Latitude*, he concludes, That the sum of the *Secants* of each point in the *Quadrant* being added successively, would exhibit a Line divided into Spaces, such as the intervals of the parallels of Latitude ought to be in a true Sea Chart, whereon the Meridians are made parallel Lines, and the *Rhombs* or Oblique Courses represented by right Lines. This is commonly known by the name of the *Meridian Line*, which though it generally be called *Mercators*, was yet undoubtedly Mr. *Wrights* Invention, (as he has made it appear in his Preface.) And the Table thereof is to be met with in most Books treating of Navigation, computed with sufficient exactness for the purpose.

It was first discovered by chance, and as far as I can learn, first publisht by Mr. *Henry Bond*, as an addition to *Norwoods Epitome* of *Navigation*, about 50 Years since, that the *Meridian Line was Analogous to a Scale of Logarithmick Tangents of half the Complements of the Latitudes*. The difficulty to prove the truth of this Proposition, seemed such to Mr. *Mercator*, the Author of *Logarithmotechnia*, that he proposed to wager a good sum of Money, against whoso would fairly undertake it, that he should not demonstrate either, that it was true or false: And about that time Mr. *John Collins*, holding a Correspondence with all the Eminent Mathematicians of the Age, did excite them to this Enquiry.

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The first that demonstrated the said *Analogy*, was the excellent Mr. *James Gregory* in his *Exercitationes Geometricæ*, published *Anno* 1668. which he did, not without a long train of Consequences and Complication of Proportions, whereby the evidence of the Demonstration is in a great measure lost, and the Reader wearied before he attain it. Nor with less work and *apparatus* hath the celebrated Dr. *Barrow* in his *Geometrical Lectures* (Lect. XI. App. I.) proved, that the *Sum* of all the *Secants* of any arch is analogous to the *Logarithm* of the *ratio* of *Radius* + *Sine* to *Rad.* — *Sine*, or which is all one, that the *Meridional parts* answering to any degree of Latitude, are as the *Logarithms* of the *rationes* of the *Versed Sines* of the distances from both the *Poles*. Since which the incomparable Dr. *Wallis* (on occasion of a paralogism committed by one Mr. *Norris* in this matter) has more fully and clearly handled this Argument, as may be seen in Num. 176. of these *Transactions*. But neither Dr. *Wallis* nor Dr. *Barrow* in their said Treatises have any where touched upon the aforesaid relation of the *Meridian-line* to the *Logarithmick Tangent* ; nor hath any one, that I know of, yet discovered the Rule for computing independently the interval of the *Meridional parts* answering to any two given Latitudes.

Wherefore having attained, as I conceive, a very facile and natural demonstration of the said Analogy, and having found out the Rule for exhibiting the *difference of Meridional parts*, between any two parallels of Latitude, without finding both the Numbers whereof they are the difference : I hope I may be entituled to a share in the emprovements of this useful part of Geometry. Desiring no other favour of some Mathematical Pretenders, than that they think fit to be so just, as neither to attribute my desire to please the Honourable the *Royal Society* in these Exercises, to any kind of *Vanity* or Love of Applause in me, (who too well know how very few these things oblige, and how small reward they procure) nor yet to complain, *coram non judice*, that I arrogate to my self the Inventions of others, and upon that pretext to depretiate what I do, unless at the same time, they can produce the Author I wrong, to prove their assertions. Such disingenuity as I have always most carefully avoided, so I wish I had not too much experience of it in the very same persons

persons, who make it their business to detract from that little share of Reputation I have in these things. But to return to the matter in hand, Let us demonstrate the following *Proposition*.

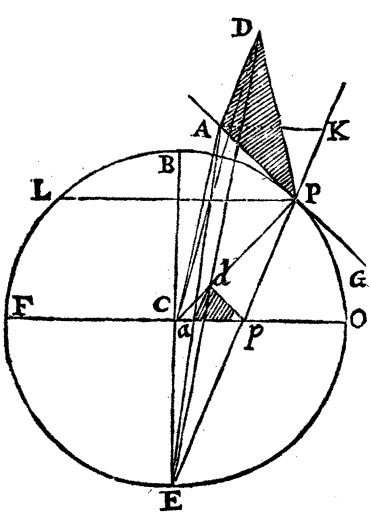
The Meridian Line is a Scale of Logarithmick Tangents of the half Complements of the Latitudes.

For this Demonstration, it is requisite to premise these four *Lemmata*.

Lemma. I. In the *Stereographick Projection* of the Sphere upon the plain of the Equinoctial, the distances from the Center, which in this case is the Pole, are laid down by the Tangents of half those distances, that is, of half the Complements of the Latitudes. This is evident from *Eucl.* 3.20.

Lem. II. In the *Stereographick Projection*, the Angles, under which the Circles intersect each other, are in all cases equal to the Spherical Angles they represent: Which is perhaps as valuable a property of this *Projection*, as that of all the Circles of the Sphere thereon appearing Circles: But this not being vulgarly known, must not be assumed without a *Demonstration*.

Let *E B P L* be any great circle of the Sphere, *E* the Eye placed in its Circumference, *C* its Center, *P* any point thereof, and let *F C O* be supposed a plain erected at right Angles to the Circle *E B P L*, on which *F C O* we design the Sphere to be projected. Draw *E P* crossing the Plain *F C O* in *p*, and *p* shall be the point *P* projected. To the point *P* draw the Tangent A P G, and on any point thereof, as A, erect a perpendicular *AD*, at right angles to the plain *EBPL*, and draw the lines *PD*, *AC*, *DC*: and the angle *APD* shall be equal to the Spherical Angle contained between the plains *APC*, *DPC*. Draw also *AE*, *DE*, intersecting



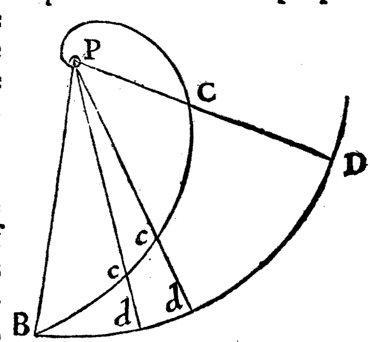
secting

fecting the plain *FCO* in the points *a* and *d*; and joyn *a d*, *p d*: I say the Triangle *a d p* is fimular to the triangle *ADP*, and the angle *ap d* equal to the angle *APD*. Draw *PL*, *AK* parallel to *FO*, and by reafon of the parallels, *a p* will be to *a d* as AK to AD: But (by *Eucl.* 3. 32.) in the triangle AKP, the angle AKP=LPE is alfo equal to APK=EPG wherefore the fides AK, AP are equal, and 'twill be, as *a p* to *a d* fo AP to AD. Whence the angles DAP, *dap* being right, the angle APD will be equal to the angle *a p d* that is, the Spherical Angle is equal to that on the Projection, and that in all Cafes. *Which was to be proved.*

This *Lemma* I lately received from Mr. *Ab. de Moivre*, though I fince underftand from Dr. *Hook* that he long ago produced the fame thing before the *Society*. However the demonftration and the reft of the difcourfe is my own.

Lemma III. On the *Globe*, the *Rhumb Lines* make equal angles with every Meridian, and by the aforegoing *Lemma*, they muft likewife make equal angles with the Meridians in the *Stereographick Projection* on the plain of the *Equator*: They are therefore, in that Projection, *Proportional Spirals* about the Pole Point.

Lemma IV. In the *Proportional Spiral* it is a known property that the angles BPC or the arches BD, are *Exponents* of the *rationes* of *BP* to *PC*: for if the arch BD be divided into innumerable equal parts, right lines drawn from them to the Center P, fhall divide the Curve BccC into an infinity of proportionals; and all the lines Pc fhall be an infinity of proportionals between PB and PC whofe number is equal to all the points d, d, in the arch BD: Whence, and by what I have delivered in Num. 216, it follows, that as BD to Bd, or as the angle BPC to the angle BPc, fo is the Logarithm of the *ratio* of PB to PC, to the Logarithm of the *ratio* of PB to Pc.



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From these *Lemmata* our Propofition is very clearly demonftrated: For by the firft, PB, Pc, PC are the *Tangents* of half the *Complements* of the *Latitudes* in the *Stereographick Projection*: And by the laft of them, the differences of Longitude, or angles at the Pole between them, are Logarithms of the *rationes* of thofe *Tangents* one to the other. But the *Nautical Meridian Line* is no other than a Table of the Longitudes, anfwering to each minute of Latitude, on the Rhumb-line making an angle of 45 degrees with the Meridian. Wherefore the Meridian Line is no other than a Scale of Logarithmick Tangents of the half Complements of the Latitudes. *Quod erat demonftrandum.*

Coroll. 1. Becaufe that in every point of any *Rhumb Line*, the difference of Latitude is to the *Departure*, as the *Radius* to the *Tangent* of the angle that Rhumb makes with the Meridian; and thofe equal *Departures* are every where to the differences of Longitude, as the Radius to the Secant of the Latitude; it follows that the differences of Longitude are, on any Rhumb, Logarithms of the fame Tangents, but of a differing *Species*; being proportioned to one another as are the Tangents of the angles made with the Meridian.

Coroll. 2. Hence any Scale of Logarithm Tangents, (as thofe of the Vulgar Tables made after *Brigg*'s form; or thofe made to *Napiers*, or any other form whatfoever) is a Table of the differences of Longitude, to the feveral Latitudes, upon fome determinate Rhumb or other: And therefore, as the Tangent of the angle of fuch *Rhumb*, to the Tangent of any other *Rhumb*: So the difference of the Logarithms of any two Tangents, to the difference of Longitude, on the propofed *Rhumb*, intercepted between the two Latitudes, of whofe half Complements you took the Logarithm Tangents.

And fince we have a very compleat Table of *Logarithm Tangents* of *Brigg*'s form, publifhed by *Vlacq*, *Anno* 1633, in his *Canon Magnus Triangulorum Logarithmicus*, computed to ten Decimal places of the Logarithm, and to every ten Seconds of the Quadrant, (which feems to be more than fufficient for the niceft Calculator) I thought fit to enquire the Oblique angle, with which that Rhumb Line croffes the Meridian, whereon the faid Canon of *Vlacq* precifely anfwers to the differences of Longitude, putting Unity for one minute thereof,

thereof, as in the Common Meridian Line. Now the *momentary augment* or *fluxion* of the Tangent Line at 45 degrees, is exactly double to the *fluxion* of the arch of the Circle, (as may easily be proved) and the Tangent of 45, being equal to *Radius*, the *fluxion* also of the Logarithm Tangent will be double to that of the arch, if the Logarithm be of *Napeirs* form : But for *Brigg's* form it will be as the same doubled arch multiplied into, 0, 43429, &c. or divided by 2, 30258, &c. Yet this must be understood only of the addition of an indivisible arch, for it ceases to be true if the arch have any determinate magnitude.

Hence it appears, that if one minute be supposed Unity, the length of the arch of one minute being ,000290888208665721596154 &c. in parts of the Radius, the proportion will be as Unity to 2,908882 &c. so Radius to the Tangent of 71° 1′ 42″ whose Logarithm is 10. 4637261172071832520 4 &c. and under that angle is the Meridian intersected by that Rumb Line, on which the *differences* of *Napeirs* Logarithm Tangents of the half Complements of the Latitudes are the true differences of Longitude, estimated in minutes and parts, taking the first Four Figures for Integers. But for *Vlacq's* Tables we must say.

As .2302585 &c. to 2908882 &c. So Radius to 1,2633114387424456921 2, &c. which is the Tangent of 51° 38′ 9″, and its Logarithm 10,1015104285077209411 62 &c. wherefore in the Rhumb Line, which makes an angle of 51° 38′ 9″ with the Meridian, *Vlacq's* Logarithm Tangents are the true differences of Longitude. And this compared with our second *Corollary* may suffice for the use of the Tables already computed.

But if a Table of Logarithm Tangents be made by extraction of the root of the Infiniteth power, whose Index is the length of the arch you put for Unity, (as for minutes the ,0002908882th &c. power) which we will call *a*; such a Scale of Tangents, shall be the true Meridian Line or sum of all the Secants taken infinitely many. Here the Reader is desired to have recourse to my little Treatise of *Logarithms*, published in N° 216. p. 58. that I may not need to repeat it. By what is there delivered, it will follow, that putting *t* for the excess or defect of any Tangent above or under the *Radius* or *Tangent* of 45; the Logarithm of the *ratio* of *Radius* to ſuch Tangent will be

$$\frac{1}{m} \text{ into } t - \tfrac{1}{2}tt + \tfrac{1}{3}ttt - \tfrac{1}{4}tttt + \tfrac{1}{5}t^5, \text{ \&c.}$$

when the arch is greater than 45gr, or

$$\frac{1}{m} \text{ into } t + \tfrac{1}{2}tt + \tfrac{1}{3}t^3 + \tfrac{1}{4}t^4 + \tfrac{1}{5}t^5, \text{ \&c.}$$

when it is leſs than 45gr. And by the ſame doctrine putting T for the Tangent of any *arch*, and *t* for the difference thereof from the Tangent of another arch, the Logarithm of their *ratio* will be

$$\frac{1}{m} \text{ into } \frac{t}{T} + \frac{tt}{2TT} + \frac{t^3}{3T^3} + \frac{t^4}{4T^4} + \frac{t^5}{5T^5}, \text{ \&c.}$$

when T is the greater Term, or

$$\frac{1}{m} \text{ into } \frac{t}{T} - \frac{tt}{2TT} + \frac{t^3}{3T^3} - \frac{t^4}{4T^4} + \frac{t^5}{5T^5}, \text{ \&c.}$$

when *T* is the leſſer Term:

And if *m* be ſuppoſed ,0002908882, &c. $= a$, its reciprocal $\frac{r}{a}$ will be, 3437,74677078493925 26, &c. which multiplied into the aforeſaid *Series*, ſhall give preciſely the difference of Meridional parts, between the two Latitudes to whoſe half complements the aſſumed Tangents belong. Nor is it material from whether Pole you eſtimate the Complements, whether the elevated or depreſſed; the Tangents being to one another in the ſame *ratio* as their Complements, but inverted.

In the ſame Diſcourſe I alſo ſhewed that the *Series* might be made to converge twice as ſwift, all the even Powers being omitted: and that putting τ for the ſum of the two Tangents the ſame Logarithm would be

$$\frac{2}{m} \text{ or } \frac{2r}{a} \text{ into } \frac{t}{\tau} + \frac{t^3}{3\tau^3} + \frac{t^5}{5\tau^5} + \frac{t^7}{7\tau^7} + \frac{t^9}{9\tau^9}, \text{ \&c.}$$

but the *ratio* of τ to *t*, or of the ſum of two Tangents to their difference, is the ſame as that of the *ſine* of the ſum of the arches, to the *ſine* of their difference. Wherefore if S be put for the *ſine* Complement of the Middle Latitude, and *s* for the *ſine* of half the difference of Latitudes, the ſame *Series* will be

$$\frac{2r}{a} \text{ into } \frac{s}{S} + \frac{s^3}{3S^3} + \frac{s^5}{5S^5} + \frac{s^7}{7S^7} + \frac{s^9}{9S^9}, \text{ \&c.}$$

wherein as the differences of Latitude are ſmaller, fewer ſteps will ſuffice. And if the Equator be put for the Middle Latitude, and conſequently $S = R$, and s to the *ſine* of the Latitude, the Meridional parts reckoned from the Equator will be

$$\frac{s}{a} + \frac{s^3}{3rra} + \frac{s^5}{5r^4a} + \frac{s^7}{7r^6a}, \text{ \&c.}$$

which is coincident with Dr. *Wallis*'s ſolution in Numb. 176. And this ſame *Series*, being half the Logarithm of the *ratio* of $R + s$ to $R - s$, that is, of the *Verſed-ſines* of the diſtances from both Poles, does agree with what Dr. *Barrow* had ſhewn in his XI. *Lecture*.

The ſame *ratio* of τ to t may be expreſſed alſo by that of *the Sum of the Co-ſines of the two Latitudes, to the ſine of their difference*: As likewiſe by that of *the Sine of the Sum of the two Latitudes, to the difference of their Co ſines*: Or by that of *the Verſed-ſine of the Sum of the Co-latitudes, to the difference of the ſines of the Latitudes*; Or as *the ſame difference of the ſines of the Latitudes, to the Verſed-ſine of the difference of the Latitudes*; all which are in the ſame *ratio* of the Co-ſine of the Middle Latitude, to the Sine of half the difference of the Latitudes. As it were eaſie to demonſtrate, if the Reader were not ſuppoſed capable to do it himſelf, upon a bare inſpection of a Sheme duly repreſenting theſe Lines.

This variety of Expreſſion of the ſame *ratio* I thought not fit to be omitted, becauſe by help of the rationality of the Sine of 30^{gr}, in all caſes where the Sum or difference of the Latitudes is 30^{gr}, 60^{gr}, 90^{gr}, 120^{gr} or 150 degrees, ſome one of them will exhibit a ſimple *ſeries*, wherein great part of the Labour will be ſaved: And beſides I am willing to give the Reader his choice which of theſe equipollent methods to make uſe of; but for his exerciſe ſhall leave the proſecution of them, and the *compendia* ariſing therefrom, to his own induſtry. Contenting my ſelf to conſider only the former, which for all uſes ſeems the moſt convenient, whether we deſign to make the whole Meridian Line, or any part thereof, *viz.*

$$\frac{2r}{a} \text{ into } \frac{s}{S} + \frac{s^3}{3S^3} + \frac{s^5}{5S^5} + \frac{s^7}{7S^7} + \frac{s^9}{9S^9}, \text{ \&c.}$$

Wherein a is the length of any Arch which you design shall be the Integer or Unity in your Meridional Parts, (whether it be a Minute, League or Degree, or any other,) S the Cosine of the Middle Latitude, and s the Sine of half the difference of Latitudes; But the Secants being the Reciprocals of the Co-sines, $\frac{s}{S}$ will be equal to $\frac{\mathfrak{f}s}{rr}$ putting $\mathfrak{f}$ for the Secant of the Middle Latitude; and $\frac{2r}{a}$ into $\frac{s}{S}$ will be $= \frac{2\mathfrak{f}s}{ar}$ This multiplied by $\frac{ss}{3SS}$ that is by $\frac{\mathfrak{ff}ss}{3rrrr}$, will give the second step: and that again by $\frac{3\mathfrak{ff}ss}{5rrrr}$, the third step; and so forward till you have compleated as many Places as you desire. But the squares of the *Sines* being in the same *ratio* with the *Versed-sines* of the double Arches, we may instead of $\frac{ss}{3SS}$ assume for our Multiplicator $\frac{v}{3V}$, or the Versed-sine of the difference of the Latitudes divided by thrice the Versed-sine of the sum of the Co-latitudes, &c. which is the utmost *Compendium* I can think of for this purpose, and the same *series* will become.

$$\frac{2sr}{aS} \text{ into } 1 + \frac{v}{3V} + \frac{v^2}{5V^2} + \frac{v^3}{7V^3} + \frac{v^4}{9V^4}$$

Hereby we are enabled to estimate the default of the method of making the Meridian line by the continual addition of the Secants of æquidifferent Archs, which as the differences of those Arches are smaller, does still nearer and nearer approach the Truth. If we assume, as Mr. *Wright* did, the Arch of one Minute to be Unity, and one Minute to be the common difference of a rank of Arches: It will be in all cases, As the Arch of one Minute, to its Chord: : So the Secant of the Middle Latitude, to the first step of our *series*. This by reason of the near equality between a and $2s$, which are to one another in the *ratio* of Unity to 1—0,0000000035256645771 3, &c. will not differ from the Secant

Secant ſ but in the ninth Figure ; being leſs than it in that proportion. The next ſtep being $+ \frac{2 ſ^3 s^3}{3 a r^5}$ will be equal to the Cube of the Secant of the middle Latitude multiplied into $\frac{2sss}{3arr}$ = 0,000000007051329087l5; which therefore unleſs the Secant exceed *ten times Radius*, can never amount to 1 in the fifth place. Theſe two ſteps ſuffice to make the Meridian Line or Logarithm Tangent to far more places than any Tables of Natural Secants yet extant, are computed to; but if the third ſtep be required it will be found to be $+ ſ^5$ into $\frac{2 s^5}{5 a r^4}$ = 0,0000000000000000089498; By all which it appears that Mr. *Wright*'s Table does no where exceed the true Meridian Parts by fully half a Minute; which ſmall difference ariſes by his having added continually the Secants of 1′,2′,3′, &c. inſtead of $0\frac{1}{2}'$, $1\frac{1}{2}$, $2'\frac{1}{2}$, $3'\frac{1}{2}$, &c. But as it is, it is abundantly ſufficient for *Nautical Uſes*. That in Sr. *Jonas Moor*'s *New Syſteme of the Mathematicks* is much nearer the Truth, but the difference from *Wright* is ſcarce ſenſible, till you exceed thoſe Latitudes where Navigation ceaſes to be practicable, the one exceeding the Truth by about half a Minute, the other being a very ſmall matter deficient therefrom.

For an example eaſie to be imitated by whoſo pleaſes, I have added the true Meridional Parts to the firſt and laſt Minutes of the Quadrant; not ſo much that there is any occaſion for ſuch accuracy, as to ſhew that I have obtained, and laid down herein, the full *Doctrine* of theſe *ſpiral Rhumbs* which are of ſo great concern in the Art of *Navigation*.

The firſt Minute is,	1,00000001410265862178
The Second,	2,00000005641063806707
The Laſt, or 89°.59′ is	30374,96343114142228643

and not 32348,5279 as Mr. *Wright* has it, by the addition of the *Secants* of every whole Minute: Nor 30249,8 as Mr. *Oughtred*'s Rule makes it, by adding the *Secants* of every other half Minute. Nor 30364,3 as Sir *Jonas Moor* had concluded it by I know not what method, tho' in the reſt of his Table he follow *Oughtred*. And

And this may suffice to shew how to derive the true Meridian Line from the Sines, Tangents or Secants supposed ready made; but we are not destitute of a Method for deducing the same independently, from the Arch it self. If the Latitude from the Equator be estimated by the length of its Arch A, Radius being Unity, and the Arch put for an *Integer* be a, as before; the Meridional Parts answering to that Latitude will be

$$\frac{1}{a} \text{ into } A + \frac{1}{6}A^3 + \frac{1}{24}A^5 + \frac{1}{\frac{84}{60}}A^7 \text{ or } \frac{61}{5040}A^7 \pm \frac{\frac{11}{2880}}{1386}A^9 \text{ or } \frac{1385}{362880}A^9, \&c.$$

which converges much swifter than any of the former *Series*, and besides has the advantage of A encreasing in Arithmetical progression, which would be of great ease, if any should undertake *de novo* to make the *Logarithm Tangents*, or the Meridian Line to many more places than now we have them. The Logarithm Tangent to the Arch of $45 + \frac{1}{2}A$ being no other than the aforesaid Series $A + \frac{1}{6}A^3 + \frac{1}{24}A^5$, &c. in *Napeirs* form, or the same multiplied into 0,43429, &c. for *Briggs*'s.

But because all these *Series* towards the latter end of the Quadrant do converge exceeding slowly, so as to render this Method almost useless, or at least very tedious. It will be convenient to apply some other Arts, by assuming the Secants of some intermediate Latitudes; and you may for s or the Sine of α the Arch of half the difference of Latitudes, substitute $\alpha - \frac{1}{6}\alpha^3 + \frac{1}{120}\alpha^5 - \frac{1}{5040}\alpha^7 + \frac{1}{362880}\alpha^9$, &c. according to Mr. *Newton*'s Rule for giving the Sine from the Arch; And if α be no more than a degree, a very few steps will suffice for all the accuracy that can be desired.

And if α be commensurable to a, that is, if it be a certain number of those Arches which you make your *Integer*, then will $\frac{\alpha}{a}$ be that number: which if we call n. the parts of the Meridional Line will be found to be.

$$\frac{sn}{r} \text{ into } \left\{ \begin{array}{l} 1 + \frac{ssaa}{3r^4} + \frac{s^4a^4}{5r^8} + \frac{s^6a^6}{7r^{12}}, \&c. \\ - \frac{aa}{6rr} - \frac{ssa^4}{6r^6} - \frac{s^4a^6}{6r^{10}}, \&c. \\ \quad + \frac{1}{120}\frac{a^4}{r^4} + \frac{13s^2a^6}{360r^8}, \&c. \\ \quad\quad - \frac{1\ a^6}{5040r^6}, \&c. \end{array} \right.$$

In this the firſt two ſteps are generally ſufficient for Nautical uſes, eſpecially when neither of the Latitudes exceed 60 degrees, and the difference of Latitudes doth not paſs 30 degrees.

But I am ſenſible I have already ſaid too much for the Learned, tho' too little for the Learner; to ſuch I can recommend no better Treatiſe, than that of Dr. *Wallis* in Numb. 176. wherein he has with his uſual brevity, and that perſpecuity peculiar to himſelf, handled this Subject from the firſt Principles, which here for the moſt part we ſuppoſe known.

I need not ſhew how, by regreſſive work, to find the Latitudes from the Meridional Parts, the Method being ſufficiently obvious. I ſhall only conclude with the propoſal of a Problem which remains to make this Doctrine compleat, and that is this.

A ſhip ſails from a given Latitude, and having run a certain number of Leagues, has altered her Longitude by a given angle. It is required to find the Courſe ſhe ſteared. The ſolution hereof would be very acceptable, if not to the publick, at leaſt to the Author of this Tract, being likely to open ſome further Light into the Myſteries of Geometry.

To Conclude, I ſhall only add, that Unity being Radius, the *Coſine* of the Arch A, according to the ſame Rules of Mr. *Newton*, will be

$$1 - \tfrac{1}{2}A^2 + \tfrac{1}{24}A^4 - \tfrac{1}{720}A^6 + \tfrac{1}{40320}A^8 - \tfrac{1}{3628800}A^{10} \ \&c.$$

from which and the former *Series* exhibiting the *Sine* by the *Arch*, by diviſion it is eaſie to conclude, that the *Natural Tangent* to the Arch A is

$$A + \frac{1}{3} A^3 + \frac{2}{15} A^5 + \frac{17}{315} A^7 + \frac{62}{2835} A^9 \text{ \&c.}$$

and the *Natural Secant* to the ſame Arch

$$1 + \frac{1}{2} A^2 + \frac{5}{24} A^4 + \frac{61}{720} A^6 + \frac{277}{8064} A^8 \text{ \&c.}$$

and from the Arithmetick of Infinites, the Number of theſe Secants being the Arch A, it follows, that the ſum Total of all the Infinite Secants on that Arch is

$$A + \frac{1}{6} A^3 + \frac{1}{24} A^5 + \frac{61}{5040} A^7 + \frac{277}{72576} A^9 \text{ \&c.}$$

the which, by what foregoes, is the *Logarithm Tangent*, of *Napeirs* form, for the Arch of $45^{gr} + \frac{1}{2} A$, as before.

And Collecting the Infinite Sum of all the *Natural Tangents* on the ſaid Arch *A*, there will arise

$$\frac{1}{2} AA + \frac{1}{12} A^4 + \frac{1}{45} A^6 + \frac{17}{2520} A^8 + \frac{31}{14175} A^{10} \text{ \&c.}$$

which will be found to be the Logarithm of the Secant of he ſame Arch *A*.

Accounts of Books.

I. *Catoptricæ & Dioptricæ Elementa, Auctore Davide Gregorio,* D. M. *Aſtronomiæ Profeſſore Saviliano Oxoniæ, & Soc. Reg. Socio,* 8°. *è Theatro Oxon.* 1695.

IN this Treatiſe the Learned Author demonſtrates the Principal Laws of Reflection and Refraction, without reſtraining himſelf to any Sect of Philoſophers; as alſo the properties of plain and ſpherical Surfaces in reflecting and refracting of Rays, and by the way ſhews how it comes that ſpherical Surfaces produce the ſame effects with thoſe of certain Spheroids and Conoids, *viz.* becauſe they have the ſame degree of Curvature. In the Catoptricks he determines the place of the Image, when the Object and the Eye are not in the ſame axis of the reflecting Sphere: an inconvenience that Dioptrical Machines are not ſubject to.

Then he proceeds to determine the ſituation and bigneſs of the Images of ſenſibly big Objects, with the quan-